

## Presymmetry in the Standard Model with adulterated Dirac neutrinos

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Recently we proposed a model for light Dirac neutrinos in which two right-handed (RH) neutrinos per generation are added to the particles of the Standard Model (SM), implemented with the symmetry of fermionic contents. The ordinary one is decoupled via the high scale type-I seesaw mechanism, while the extra pairs off with its left-handed (LH) partner. The symmetry of lepton and quark contents was merely used as a guideline to the choice of parameters because it is not a proper symmetry. Here we argue that the underlying symmetry to take for this correspondence is presymmetry, the hidden electroweak symmetry of the SM extended with RH neutrinos defined by transformations which exchange lepton and quark bare states with the same electroweak charges and no Majorana mass terms in the underlying Lagrangian. It gives a topological character to fractional charges, relates the number of families to the number of quark colors, and now guarantees the great disparity between the couplings of the two RH neutrinos. Thus, Dirac neutrinos with extremely small masses appear as natural predictions of presymmetry, satisfying the 't Hooft's naturalness conditions in the extended seesaw where the extra RH neutrinos serve to adulterate the mass properties in the low scale effective theory, which retains without extensions the gauge and Higgs sectors of the SM. However, the high energy threshold for the seesaw implies new physics to stabilize the quantum corrections to the Higgs boson mass in agreement with the naturalness requirement.

*Keywords:* Dirac neutrinos; extra right-handed neutrinos; seesaw mechanism; presymmetry.

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### 1. Introduction

The nonzero mass of neutrinos is one of the most compelling evidences for physics beyond the Standard Model (SM) based on the gauge groups  $SU(3)_c \times SU(2)_L \times U(1)_Y$  and Higgs fields in a single doublet.<sup>1–3</sup> In the SM, neutrinos do not have Dirac mass as in the case of charged leptons and quarks because only left-handed (LH) neutrinos are included. They do not possess Majorana mass either, as  $B - L$  is an (accidental) exact global symmetry of the SM, where  $B$  and  $L$  denote the baryon and lepton numbers.

In order to produce generic mass terms, one must then extend the SM by adding three right-handed (RH) neutrinos to arrange Dirac mass terms and also break the  $B - L$  symmetry through RH Majorana mass terms, which are allowed by the

gauge symmetry of the SM. It is a minimal expansion where the gauge and Higgs sectors of the SM are maintained. Invoking the naturalness criterion of 't Hooft,<sup>4</sup> this breakdown may be small since Majorana mass terms equal to zero recover the  $B - L$  symmetry and the model becomes more symmetric. Actually, this is the well-known pseudo-Dirac scenario,<sup>5–9</sup> where the dominant contribution to neutrino masses comes from Dirac mass terms with small corrections from Majorana mass terms. It appears as an alternative to the popular seesaw approach,<sup>10–13</sup> where neutrino mass terms are preponderantly of Majorana type, assuming a high scale of new physics which leads to very different masses for light LH and heavy RH neutrinos. This scenario also makes natural in the sense of 't Hooft the tiny mass of neutrinos since the symmetry of lepton number is restored in the limit of small mass terms equal to zero.

Thus far, the problem on the Dirac or Majorana nature of massive neutrinos remains unresolved. As a matter of fact, no signals in the search for neutrinoless double-beta decay of nuclei have been observed,<sup>14–16</sup> which is at present the most feasible process capable of establishing the Majorana nature of neutrinos. Hence, light neutrinos can be Dirac-like fermions. In this work we restrict ourselves to this plausible possibility, which is not so much explored in the literature.

Within the framework of the SM just extended with three RH neutrinos, there is no known natural mechanism to accounting for the smallness of Dirac neutrino masses in comparison with the charged lepton masses. The inclusion of small Majorana mass terms, in addition to Dirac mass terms, as in the pseudo-Dirac scenario, does not explain why neutrino masses are so light compared to those of charged leptons. The inclusion of heavy Majorana mass terms, as done in the type-I seesaw mechanism, only leads to light Majorana neutrinos, but not to Dirac neutrinos. Hence, explaining the Dirac nature of light neutrinos requires more physics, beyond adding three RH neutrinos with small or large Majorana mass terms. In other words, in the extension of the SM where a RH neutrino per generation is introduced, the small value of the Dirac mass of neutrinos compared to the charged leptons is not natural, in the sense of the 't Hooft's principle of naturalness, so that a fine-tuning is badly needed.

On the other hand, the current experimental status magnifies the disturbing possibility that the successes of the SM may continue at the TeV scale, so that the new runs of the Large Hadron Collider (LHC) do not produce any significant hint of the new physics which introduces extra gauge and Higgs fields with breaking scales at the TeV range. If this simple view of the experimental situation is adopted, one is left with the SM and the presumed Dirac neutrinos with extremely small masses, for which there is no explanation. This scenario can be expanded to accommodate dark matter, for instance, by introducing extra sterile neutrinos, although this point is not addressed here.

Even assuming this very conservative scenario, however, we proposed for the first time in Ref. 17 a simple approach to understanding the small masses of Dirac neutrinos in comparison with charged leptons where masses are adulterated by

adding in each generation of the SM a second, almost inert, RH neutrino with small Majorana mass. It is a minimal extension in which the gauge and Higgs fields of the SM are kept, consistent with the fact that the experimental evidence at the TeV scale may still support the physics of the SM with massive neutrinos of Dirac type. It applies the seesaw mechanism to suppress the ordinary RH states with heavy Majorana masses and uses the surviving extra RH neutrinos to generate the tiny Dirac masses, i.e. the actual nature of light neutrinos would be of adulterated Dirac type, where the usual RH neutrino is replaced by the extra one of much smaller couplings. The key ingredient of this model to making natural the large difference between the Majorana masses of the two RH neutrinos in each generation is the correspondence between lepton and quark contents when one of the RH neutrinos is introduced in each generation. But this relation of particle contents is not a proper symmetry to invoking the 't Hooft's principle of naturalness, i.e. there is no symmetry transformation between leptons and quarks that maintains the Lagrangian in the model invariant as they have different charges and Majorana mass terms in the quark sector are absent. Therefore such a correspondence was regarded just as a guideline to the choice of parameters, conceding that the proper symmetry behind it should be founded within the SM with RH neutrinos itself.

From another standpoint, the SM extended with three RH Dirac neutrinos has been considered to re-establishing the electroweak lepton-quark symmetry and incorporate presymmetry,<sup>18,19</sup> the symmetry hidden by the nontrivial topology of the weak gauge fields which goes with the lepton-quark symmetry from weak to electromagnetic interactions, where a symmetry of lepton and quark contents is demanded. More specifically, the symmetric electroweak patterns have been explained in terms of underlying bare states of leptons and quarks having the same charges and no Majorana mass terms, but located in a topologically-nontrivial vacuum of the weak gauge fields in a manner that the charge shifts are induced, in theory, via vacuum tunneling weak instantons. Consequently, fractional charges get a topological character and the number of families becomes associated with the number of quark colors. And presymmetry transformations exchange the bare states of leptons and quarks keeping the underlying Lagrangian invariant.

Our aim in this work is to build up a consistent model for light Dirac-type neutrinos, establishing presymmetry as the underlying symmetry required to substantiate the symmetry of lepton and quark contents used in the SM extended with two RH neutrinos per generation, first discussed in Ref. 17. This unifies models of neutrinos and presymmetry, showing that Dirac-like neutrinos with masses exceptionally small compared to charged leptons are natural predictions of presymmetry, in the sense of 't Hooft. As described above, all of these motivated by the successes of the SM well above the TeV scale and the actual, though not so well explored possibility that light neutrinos have a Dirac nature.

Specifically, the motivation for presymmetry is the finding of a proper symmetry which distinguishes the two RH neutrinos. Allowing this concrete symmetry entails that the Dirac and Majorana mass terms of the extra RH neutrinos are not

free parameters, independent of the original RH neutrinos. And they are not made small by fine-tuning. Their smallness compared to the original RH neutrinos are guaranteed by the presymmetry imposed on the theory with these ones at the high-energy seesaw scale, much heavier than the electroweak symmetry breaking scale. The 't Hooft's argument of naturalness for the small values of the Dirac and Majorana masses of the extra RH neutrinos in the Lagrangian relies on this presymmetry with the usual RH neutrinos; as the couplings of the extra RH neutrinos tend to zero, the underlying theory only involving the original RH neutrinos becomes more symmetric. This symmetry guarantees the quantum corrections of such parameters to be proportional to the parameters themselves and its interplay with the seesaw mechanism leading to the low-energy effective theory with the original RH neutrinos decoupled only introduces omissible tiny corrections to the mass parameters. In particular, the smallness of the Dirac mass with the extra RH neutrino in comparison with the charged leptons appears robust.

Yet, there is no natural protection of the Higgs boson mass of the SM against the large quantum corrections introduced by the high scale of the seesaw for neutrino masses. The problem of naturalness arises from the disparity between the energy scales for the seesaw threshold and its upper value allowed by the natural condition.<sup>20</sup> In order to maintain the stability of the Higgs mass, the new physics associated with the seesaw must then suppress the new contributions. The only best known manner of having this cancellation in agreement with the naturalness requirement is through supersymmetry. What is more, it can be realized partially, as recently explored in Ref. 21, where the SM is considered non-supersymmetric in the first stage. The implementation of our extended seesaw with the new physics able to control the quantum corrections to the Higgs mass, however, is beyond the scope of this work.

The paper is organized as follows. In Sec. 2, we recall for completeness and subsequent discussion the needed results about neutrino masses in the mixed scenario of Dirac and Majorana neutrinos where two RH neutrinos per generation are added. In Sec. 3, we look into the SM expanded with the adulterated Dirac neutrinos of naturally small masses, where ordinary RH neutrinos are decoupled. In Sec. 4, we go over presymmetry in the extension of the SM with RH neutrinos and its relevance in the model of light Dirac neutrinos. In Sec. 5, we refer to phenomenological implications of the unified model of massive neutrinos and presymmetry. The conclusions are summarized in Sec. 6.

## 2. Generic Neutrino Masses with Two Right-Handed Neutrinos per Generation

The addition of RH neutrinos and the violation of lepton number conservation are modifications of the SM in order to produce neutrino masses in a generic way. Its expansion with two RH neutrinos in each generation is to construct light Dirac neutrinos with the extra one. In the following we review, for completeness and

subsequent discussion, the basic results for the SM extended with two RH neutrinos per generation, preserving its gauge and Higgs structures.<sup>17</sup> The first RH neutrino is the ordinary one, which may carry a  $B - L$  charge and form a doublet with its RH charged lepton partner, as in models of left-right symmetry.<sup>22–24</sup> The other is a secondary singlet with generally small couplings and no local charges. The crucial element of the model is the symmetry of lepton and quark contents when just one of the RH neutrinos is added. In this manner, invoking the 't Hooft's criterion,<sup>4</sup> the smallness of couplings of the second RH neutrino appears natural since the symmetry of quark and lepton contents with the first RH neutrino is re-established if these couplings are set to be zero.

The symmetry of fermionic contents, however, is merely used as a guideline to the choice of parameters because it is not a proper symmetry in the Lagrangian. This means that one cannot define a symmetry transformation between leptons and quarks to keep the Lagrangian invariant, as these particles have different charges and Majorana mass terms are not present in the quark sector. In Sec. 4, we discuss on the proper symmetry that must be attached to this symmetry of lepton and quark contents when one of the RH neutrinos is introduced for each generation.

The mass terms after spontaneous electroweak symmetry breaking are<sup>17</sup>

$$-\mathcal{L}_\nu = \frac{1}{2} \begin{pmatrix} \bar{\nu}_L & \bar{\nu}_L^c & \bar{\nu}_L^c \end{pmatrix} \begin{pmatrix} 0 & M_D & M'_D \\ M_D^T & M_R & M'^T \\ M'^T_D & M' & M'_R \end{pmatrix} \begin{pmatrix} \nu_R^c \\ \nu_R \\ \nu'_R \end{pmatrix} + h.c., \quad (1)$$

where  $\nu_R$  ( $\nu'_R$ ) is a three-component vector denoting the ordinary (extra) RH neutrinos, and  $M_D$ ,  $M'_D$  are  $3 \times 3$  complex matrices referring to the Dirac mass terms,  $M_R$ ,  $M'_R$  to the RH Majorana mass terms, and  $M'$  to the mixing terms; the phase convention is  $\nu_R^c = C\bar{\nu}_L^T$ .

The values of masses and couplings of RH neutrinos should be understood. Because they are not part of the SM, whose origin in turn is not known yet, the form of their findings may not be well defined. In the SM every LH charged lepton and quark has its RH charged lepton or quark partner, while the RH partner of the neutrino is absent. This content of chiral fermions is clearly not symmetric. The simplest manner of having such a symmetry between leptons and quarks is through the introduction of ordinary RH neutrinos,  $\nu_R$ .<sup>25,26</sup> Within the formalism of Eq. (1), it corresponds to  $M'_D = 0$ ,  $M' = 0$  and  $M'_R = 0$ , but keeping  $M_D$  and  $M_R$  as nontrivial. Here the proposal embraces the rationale of the type-I seesaw mechanism based on the assumptions that  $M_D$  has the same mass scale as charged leptons, and  $M_R$  is sufficiently large to suppress  $M_D$  according to  $M_D M_R^{-1} M_D^T$ . Thus, the lepton-quark symmetry of particle content together with a large  $M_R$  and  $M'_D = M' = M'_R = 0$  mimic a high scale type-I seesaw, decoupling the ordinary RH neutrinos.

The lepton-quark correspondence, however, is broken when the extra RH neutrinos,  $\nu'_R$ , are added. This is regarded as a reason for having small couplings  $M'_D$ ,  $M'$ ,  $M'_R$  for  $\nu'_R$ , as the 't Hooft's naturalness criterion applied to this symmetry of lepton

and quark contents in the Lagrangian gives a ready explanation. Indeed these extra RH neutrinos establish an alternative, exceptionally weak, lepton–quark correspondence. Therefore the symmetry of fermionic contents distinguishes  $\nu_R$  from  $\nu'_R$  by requiring a large difference between  $M_D$ ,  $M_R$  and  $M'_D$ ,  $M'_R$ , respectively, which parametrize the two forms of the symmetry. This other lepton–quark symmetry of particle content together with  $M_D = M' = M_R = 0$  and small  $M'_R$  mimic a low scale pseudo-Dirac scenario, but pairing off the LH with the extra RH neutrinos. As emphasized above, the correspondence of leptons and quarks is only considered as a guideline to coupling selections since it is not a proper symmetry in the electroweak Lagrangian.

The mass matrix in Eq. (1) is diagonalizable by the unitary transformation

$$\mathcal{U}^\dagger \mathcal{M} \mathcal{U}^* = \begin{pmatrix} D_L & 0 & 0 \\ 0 & D_R & 0 \\ 0 & 0 & D'_R \end{pmatrix}, \quad (2)$$

where  $D_L$ ,  $D_R$ , and  $D'_R$  are diagonal, real, and non-negative  $3 \times 3$  matrices. The unitary matrix can be written as

$$\mathcal{U}^\dagger = \begin{pmatrix} V_L^\dagger & 0 & 0 \\ 0 & V_R^\dagger & 0 \\ 0 & 0 & V_R'^\dagger \end{pmatrix} \begin{pmatrix} (\frac{1}{\sqrt{2}}I + W_{LL}^\dagger) & V_{RL}^\dagger & (-\frac{1}{\sqrt{2}}I + W_{RL}'^\dagger) \\ V_{LR}^\dagger & I & V_{RL}'^\dagger \\ (\frac{1}{\sqrt{2}}I + W_{LR}'^\dagger) & V_{LR}'^\dagger & (\frac{1}{\sqrt{2}}I + W_{RR}'^\dagger) \end{pmatrix}, \quad (3)$$

where  $V_L$ ,  $V_R$ , and  $V_R'$  are unitary  $3 \times 3$  complex matrices. If it is assumed that  $M_R$  and  $M'_D$  are non-singular and symmetric matrices, and that  $M'_R$ ,  $M'$ ,  $M'_D$ ,  $M_D \ll M_R$  and  $M'_R$ ,  $M_D M_R^{-1} M_D^T$ ,  $M' M_R^{-1} M'^T$ ,  $M' M_R^{-1} M_D^T \ll M'_D$ , as argued above, and the constraints from unitarity and the matrix  $\mathcal{M} \mathcal{U}^*$  are used as in the ordinary seesaw mechanism, the following expressions are obtained:

$$\begin{aligned} W_{LL}^\dagger &\simeq \frac{1}{4\sqrt{2}} M'_R M_D'^{-1} + \frac{1}{4\sqrt{2}} (M_D - M') M_R^{-1} (M_D^T + M'^T) M_D'^{-1}, \\ W_{RR}'^\dagger &\simeq \frac{1}{4\sqrt{2}} M'_R M_D'^{-1} + \frac{1}{4\sqrt{2}} (M_D + M') M_R^{-1} (M_D^T - M'^T) M_D'^{-1}, \\ W_{RL}'^\dagger &\simeq W_{LL}^\dagger, \\ W_{LR}'^\dagger &\simeq -W_{RR}'^\dagger, \\ V_{RL}^\dagger &\simeq -(\frac{1}{\sqrt{2}}I + W_{LL}^\dagger) M_D M_R^{-1} + (\frac{1}{\sqrt{2}}I - W_{LL}^\dagger) M' M_R^{-1}, \\ V_{LR}'^\dagger &\simeq -(\frac{1}{\sqrt{2}}I - W_{RR}'^\dagger) M_D M_R^{-1} - (\frac{1}{\sqrt{2}}I + W_{RR}'^\dagger) M' M_R^{-1}, \\ V_{LR}^\dagger &\simeq M_R^{-1\dagger} M_D^\dagger, \\ V_{RL}'^\dagger &\simeq M_R^{-1\dagger} M'^\dagger. \end{aligned} \quad (4)$$

These lead to

$$\begin{aligned}
 D_L &\simeq V_L^\dagger [-M'_D + \frac{1}{2}M'_R - \frac{1}{2}(M_D - M')M_R^{-1}(M_D^T - M'^T)]V_L^* \\
 &\simeq -V_L^\dagger M'_D V_L^*, \\
 D_R &\simeq V_R^\dagger M_R V_R^*, \\
 D'_R &\simeq V_R'^\dagger [M'_D + \frac{1}{2}M'_R - \frac{1}{2}(M_D + M')M_R^{-1}(M_D^T + M'^T)]V_R'^* \\
 &\simeq V_R'^\dagger M'_D V_R'^*.
 \end{aligned} \tag{5}$$

In the mixed pseudo-Dirac and seesaw regimes with  $M'_R = 0$  and  $M_D$ ,  $M'$  kept down, there are three light almost degenerate pairs of mass eigenstates with small mass differences, having almost maximal mixing of LH neutrinos  $\nu_L$  and adulterant RH neutrinos  $\nu'_R$ , and three heavy, mostly ordinary RH neutrinos  $\nu_R$  with mass matrix  $M_R$ . The mass of light neutrinos are of the order of  $M'_D$  instead of  $M_D$ , which are down by the seesaw mechanism. The incidence of matrices  $V_{LR}$ ,  $V_{RL}$ ,  $V'_{LR}$ , and  $V'_{RL}$  are reduced by  $M_R$ , while  $W_{LL}$ ,  $W'_{RR}$ ,  $W'_{LR}$ , and  $W'_{RL}$  are by  $M_R$  and/or  $M'_D$ .

### 3. The Standard Model with Adulterated Dirac Neutrinos

The RH neutrinos with large masses can be integrated out by means of the equation of motion

$$\frac{d\mathcal{L}_\nu}{d\nu_R} = 0, \tag{6}$$

which gives

$$\bar{\nu}_L^c = -\bar{\nu}_L M_D M_R^{-1} - \bar{\nu}_L' M' M_R^{-1}, \quad \nu_R = -M_R^{-1} M_D^T \nu_R^c - M_R^{-1} M'^T \nu_R'. \tag{7}$$

The effective Lagrangian is then written as

$$-\mathcal{L}_\nu = \frac{1}{2} \begin{pmatrix} \bar{\nu}_L & \bar{\nu}_L^c \end{pmatrix} \begin{pmatrix} M_{LL} & M'_{LR} \\ M'_{LR}^T & M'_{RR} \end{pmatrix} \begin{pmatrix} \nu_R^c \\ \nu_R' \end{pmatrix} + h.c., \tag{8}$$

where

$$\begin{aligned}
 M_{LL} &\simeq -M_D M_R^{-1} M_D^T, & M'_{LR} &\simeq M'_D - M_D M_R^{-1} M'^T, \\
 M'_{RR} &\simeq -M' M_R^{-1} M'^T.
 \end{aligned} \tag{9}$$

The mass matrix in Eq. (8) is diagonalized by the approximately unitary matrix

$$\mathcal{U}^\dagger \simeq \begin{pmatrix} V_L^\dagger & 0 \\ 0 & V_R'^\dagger \end{pmatrix} \begin{pmatrix} (\frac{1}{\sqrt{2}}I + W_{LL}^\dagger) & (-\frac{1}{\sqrt{2}}I + W_{LL}^\dagger) \\ (\frac{1}{\sqrt{2}}I - W_{RR}^\dagger) & (\frac{1}{\sqrt{2}}I + W_{RR}^\dagger) \end{pmatrix}, \tag{10}$$

such that

$$\mathcal{U}^\dagger \mathcal{M} \mathcal{U}^* = \begin{pmatrix} D_L & 0 \\ 0 & D'_R \end{pmatrix}, \tag{11}$$

where  $W_{LL}^\dagger$  and  $W_{RR}^\dagger$ ,  $D_L$  and  $D'_R$  have the expressions given in Eqs. (4) and (5) with  $M'_R = 0$ .

Now, the mass hierarchies

$$M_D M_R^{-1} M_D^T, M_D M_R^{-1} M'^T, M' M_R^{-1} M'^T \ll M'_D \ll M_D \ll M_R, \quad (12)$$

lead to the mass matrix

$$\mathcal{M} \simeq \begin{pmatrix} 0 & M'_D \\ M_D^T & 0 \end{pmatrix}, \quad (13)$$

which is consistent with the SM extended with the extra RH neutrinos,  $\nu'_R$ , having a Dirac nature. Actually, in the vanishing limit of the small values  $M_D M_R^{-1} M_D^T$ ,  $M_D M_R^{-1} M'^T$ ,  $M' M_R^{-1} M'^T$ , a lepton number conservation and a lepton–quark symmetry are set up at low energies. It is the adulterated lepton–quark symmetry of particle content in terms of  $\nu'_R$  with all couplings of  $\nu_R$  removed ( $M_D = M_R = M' = 0$ ). Dirac neutrino masses much smaller than those of charged leptons now appear natural because  $M'_D = 0$  (with  $M' = M'_R = 0$ ) restores an enhanced symmetry in the original Lagrangian, specifically, the symmetry of lepton and quark contents including the ordinary neutrino partners  $\nu_R$ . Thus adulterated Dirac neutrinos with tiny masses can be accommodated naturally. Once more, we emphasize that the correspondence between lepton and quark contents just serves as a guideline to the choice of parameters since it is not a proper symmetry in the electroweak Lagrangian.

#### 4. Presymmetry in the Standard Model with Right-Handed Neutrinos

We now substantiate the lepton–quark symmetry of particle content through a proper symmetry of the SM extended with RH neutrinos, based on the gauge groups  $SU(3)_c \times SU(2)_L \times U(1)_Y$  and Higgs fields in a doublet. An available simple option for this symmetry is presymmetry, where the crucial elements to have a well-defined symmetry transformation appear naturally. In fact, presymmetry is conceived as a symmetry of an electroweak Lagrangian under transformations on underlying bare states of leptons and quarks having the same charges and no associated Majorana mass terms. To make the whole in a consistent way, we review in the following the key arguments for presymmetry when one RH neutrino is added in each generation.

On the one hand, there is the following hypercharge symmetry between chiral leptons and quarks within each of their three families:<sup>18,19</sup>

$$\begin{aligned} Y(\nu_L) &= Y(u_L) + \Delta Y(u_L) = -1, & Y(e_L) &= Y(d_L) + \Delta Y(d_L) = -1, \\ Y(\nu_R) &= Y(u_R) + \Delta Y(u_R) = 0, & Y(e_R) &= Y(d_R) + \Delta Y(d_R) = -2, \end{aligned} \quad (14)$$

and, on the other hand,

$$\begin{aligned} Y(u_L) &= Y(\nu_L) + \Delta Y(\nu_L) = \frac{1}{3}, & Y(d_L) &= Y(e_L) + \Delta Y(e_L) = \frac{1}{3}, \\ Y(u_R) &= Y(\nu_R) + \Delta Y(\nu_R) = \frac{4}{3}, & Y(d_R) &= Y(e_R) + \Delta Y(e_R) = -\frac{2}{3}, \end{aligned} \quad (15)$$



with the  $\Delta Y$  equal to  $4/3$  for leptons and  $-4/3$  for quarks being related to the lepton and baryon numbers according to

$$\Delta Y = \frac{4}{3} (L - 3B), \quad (16)$$

where the conventional relation  $Q = T_3 + Y/2$  between electric charge, weak isospin, and hypercharge, is used. Any other hypercharge normalization can modify the value of the global fractional piece  $\Delta Y$ , but the charge symmetry described in Eqs. (14) and (15) is preserved.<sup>27</sup>

Presymmetry is associated with the equality of lepton and quark charges when the global part  $\Delta Y$  is set apart. The inclusion of RH neutrinos is essential to completing this correspondence between charges, which in turn allows a symmetry of lepton and quark contents when one RH neutrino per generation is introduced; the symmetric pattern in terms of the extra, adulterant RH neutrinos considers  $\nu'_R$  instead of  $\nu_R$  in Eqs. (14) and (15).

The charge symmetry and charge dequantization underlying Eqs. (14) and (15) can be understood if prelepton and prequark states are taken into account. These are defined by the quantum numbers of leptons and quarks, respectively, except charge values, and denoted by a hat accent over the corresponding flavor symbol. The hypercharges of preleptons and prequarks are the same as their respective quark and lepton weak partners. From Eqs. (14) and (15) one is then led, in the first case, to<sup>18,19</sup>

$$\begin{aligned} Y(\nu_L) &= Y(\hat{\nu}_L) + \Delta Y(\hat{\nu}_L), & Y(e_L) &= Y(\hat{e}_L) + \Delta Y(\hat{e}_L), \\ Y(\nu_R) &= Y(\hat{\nu}_R) + \Delta Y(\hat{\nu}_R), & Y(e_R) &= Y(\hat{e}_R) + \Delta Y(\hat{e}_R), \end{aligned} \quad (17)$$

and, in the other case, to

$$\begin{aligned} Y(u_L) &= Y(\hat{u}_L) + \Delta Y(\hat{u}_L), & Y(d_L) &= Y(\hat{d}_L) + \Delta Y(\hat{d}_L), \\ Y(u_R) &= Y(\hat{u}_R) + \Delta Y(\hat{u}_R), & Y(d_R) &= Y(\hat{d}_R) + \Delta Y(\hat{d}_R), \end{aligned} \quad (18)$$

with prelepton–quark charge symmetry established as

$$\begin{aligned} Y(\hat{\nu}_L) &= Y(u_L), & \Delta Y(\hat{\nu}_L) &= \Delta Y(u_L), \\ Y(\hat{\nu}_R) &= Y(u_R), & \Delta Y(\hat{\nu}_R) &= \Delta Y(u_R), \\ Y(\hat{e}_L) &= Y(d_L), & \Delta Y(\hat{e}_L) &= \Delta Y(d_L), \\ Y(\hat{e}_R) &= Y(d_R), & \Delta Y(\hat{e}_R) &= \Delta Y(d_L), \end{aligned} \quad (19)$$

and prequark–lepton charge symmetry given by

$$\begin{aligned}
Y(\hat{u}_L) &= Y(\nu_L), & \Delta Y(\hat{u}_L) &= \Delta Y(\nu_L), \\
Y(\hat{u}_R) &= Y(\nu_R), & \Delta Y(\hat{u}_R) &= \Delta Y(\nu_R), \\
Y(\hat{d}_L) &= Y(e_L), & \Delta Y(\hat{d}_L) &= \Delta Y(e_L), \\
Y(\hat{d}_R) &= Y(e_R), & \Delta Y(\hat{d}_R) &= \Delta Y(e_L),
\end{aligned} \tag{20}$$

where the relation of  $\Delta Y$  with the lepton and baryon numbers for preleptons and prequarks is now

$$\Delta Y = \frac{4}{3} (3L - B), \tag{21}$$

with  $\Delta Y$  equal to  $-4/3$  for preleptons and  $4/3$  for prequarks. Thus,  $L = -1/3$  for preleptons, with the 3 being attributed to the number of families, and  $B = -1$  for prequarks.<sup>19</sup>

The underlying lepton–quark charge symmetry when just one RH neutrino is introduced for each generation has been set forth with the global piece of hypercharge having a weak topological character. It has been argued that the fact that any weak topological property cannot have observable effects at the zero-temperature scale because of the smallness of the weak coupling, implies that the charge structures reflected in Eqs. (17) and (18) do not apply to physical leptons and quarks, but to new underlying states referred to as bare leptons and quarks which have topological ingredients.<sup>19</sup> However, the assignments of these bare leptons and quarks to the gauge groups of the SM are the same of standard leptons and quarks. The electroweak presymmetry is therefore between preleptons and bare quarks, and between prequarks and bare leptons. Due to their topological properties, preleptons and bare quarks have also been named topological preleptons and topological quarks, respectively.

The interactions of topological preleptons and topological quarks, as well as of prequarks and bare leptons, with the gauge fields are supposed to be defined by the same Lagrangian of the gauge sector of the SM with leptons and quarks excepting hypercharge couplings. In the Yukawa sector, Majorana mass terms are forbidden for RH preneutrinos since these have nonzero hypercharge, but they are a possibility at the physical lepton–quark level. Presymmetry is the invariance of the bare electroweak Lagrangian under flavor transformations of a  $Z_2$  group which exchange topological preleptons and topological quarks on the one hand,  $\hat{\nu}_{L(R)} \leftrightarrow u_{L(R)}$ ,  $\hat{e}_{L(R)} \leftrightarrow d_{L(R)}$ , and prequarks and bare leptons on the other hand,  $\hat{u}_{L(R)} \leftrightarrow \nu_{L(R)}$ ,  $\hat{d}_{L(R)} \leftrightarrow e_{L(R)}$ .

The charge shifts are originated by the nonstandard hypercharges of the new fermionic states, which produce gauge anomalies in the couplings by fermion triangle loops of three currents related to the chiral  $U(1)_Y$  and  $SU(2)_L$  gauge symmetries. In fact, in the scenario of topological preleptons and quarks, for example, the  $U(1)_Y$

gauge current in all representations

$$\hat{J}_Y^\mu = \bar{\ell}_L \gamma^\mu \frac{Y}{2} \hat{\ell}_L + \bar{\ell}_R \gamma^\mu \frac{Y}{2} \hat{\ell}_R + \bar{q}_L \gamma^\mu \frac{Y}{2} q_L + \bar{q}_R \gamma^\mu \frac{Y}{2} q_R, \quad (22)$$

exhibits the  $U(1)_Y [SU(2)_L]^2$  and  $[U(1)_Y]^3$  anomalies due to the nonvanishing of the following sums which include one RH preneutrino per generation:

$$\sum_L Y = 8, \quad \sum_{LR} Y^3 = -24, \quad (23)$$

where the first runs over the LH and the second over the LH and RH topological preleptons and quarks, with  $(-1)$  for the RH contributions. Their cancellations need a counterterm which contains topological currents or Chern–Simons classes associated with the  $U(1)_Y$  and  $SU(2)_L$  gauge groups:

$$J_T^\mu = \frac{1}{4} K^\mu \sum_L Y + \frac{1}{16} L^\mu \sum_{LR} Y^3 = 2 K^\mu - \frac{3}{2} L^\mu, \quad (24)$$

where

$$K^\mu = \frac{g^2}{8\pi^2} \epsilon^{\mu\nu\lambda\rho} \text{tr} \left( W_\nu \partial_\lambda W_\rho - \frac{2}{3} ig W_\nu W_\lambda W_\rho \right), \quad (25)$$

$$L^\mu = \frac{g'^2}{12\pi^2} \epsilon^{\mu\nu\lambda\rho} A_\nu \partial_\lambda A_\rho,$$

so that the new current  $J_Y^\mu = \hat{J}_Y^\mu + J_T^\mu$  is anomaly free, gauge noninvariant, and also symmetric under the exchange of topological preleptons and quarks. Moreover, its charge is not conserved due to the topological charge which gives the change in the topological winding number of the asymptotic, pure gauge field configurations, assuming that the space–time region of nonzero energy density is bounded. Indeed, advocating the principle of equality for all preleptons of the system in the partition of the topological charge,<sup>18</sup> the change in each charge, using Eqs. (24) and (25) for the pure gauge fields, is

$$\Delta Q_Y = \frac{1}{6} (n_+ - n_-) = \frac{1}{6} n, \quad (26)$$

with the topological charge given by

$$n = \int d^4x \partial_\mu K^\mu = \frac{g^2}{16\pi^2} \int d^4x \text{tr} (W_{\mu\nu} \tilde{W}^{\mu\nu}). \quad (27)$$

These topological numbers vanish in the  $U(1)_Y$  case.

Vacuum states labeled by different topological numbers are then tunneled by  $SU(2)_L$  instantons carrying topological charges, making possible in principle the charge shifts and transitions from fermions with nonstandard to those with standard charges. Each hypercharge is changed by a same amount:

$$Y(\hat{\ell}) \rightarrow Y(\hat{\ell}) + \frac{n}{3}. \quad (28)$$

The value  $n = -4$  leading to Eq. (21) is set by the cancellation of anomalies and elimination of the associated counterterm (see Eq. (24)), demanded by the gauge

invariance and renormalizability of the theory; the coefficient  $3L - B$  in Eq. (21) is just a counting number.<sup>19</sup>

According to the presymmetry model, topological preleptons have a vacuum gauge field configuration of winding number  $n_- = 4$ , if gauge freedom is used to set  $n_+ = 0$  for that of leptons. The transformation of topological preleptons into bare leptons is via a Euclidean topological weak instanton with topological charge  $n = -4$ , conceived in Minkowski space-time as a quantum mechanical tunneling event between vacuum states of weak  $SU(2)_L$  gauge fields with different topological winding numbers. Thus, topological preleptons and bare leptons are differentiated by the topological vacua of their weak gauge configurations, tunneled by a weak four-instanton bearing the topological charge and inducing the global fractional piece of charge required for normalization.

However, the transitions from topological preleptons to bare leptons by means of the weak  $SU(2)_L$  instantons, as well as those from prequarks to topological quarks, do not occur in the actual world because topological preleptons, prequarks and topological quarks are not physical dynamical entities. They are bare states of leptons and quarks considered as mathematical entities out of which the observed particle states are constructed. In a sense, such transformations are frustrated by the extreme smallness of the instanton transition probability at zero temperature, and the charge normalization eliminates the extraordinarily large time scale for them, leading to leptons and quarks with trivial topology and standard charges. The replacement of bare leptons and quarks with normalized charges by the standard ones in the electroweak Lagrangian is straightforward as they have the same quantum numbers,<sup>18,19</sup> and the insertion of Majorana mass terms for RH neutrinos gives the shape of the extended effective theory.

Despite this, one still has a proper symmetry transformation at the level of preleptons defined by the exchange of all topological preleptons and quarks in the electroweak sector of the Lagrangian, which requires a correspondence between fermionic contents at the stages of preleptons and leptons. But this is precisely what is needed to have a natural framework which allows light Dirac-like neutrinos in the low-energy theory. In fact, as seen in Secs. 2 and 3, the addition of a second RH neutrino per generation coupling à la Dirac to the LH neutrino provides the seed for that as the relation between light neutrino masses and the charged lepton masses is broken. By means of this, the naturalness problem is solved, i.e. the question why light Dirac neutrinos are so much lighter than charged leptons is answered.

The well-defined presymmetry now validates the sequence of hierarchies assumed in Secs. 2 and 3. The first hierarchy,  $M_D \ll M_R$ , mimics the standard high scale type-I seesaw scenario in which only one RH neutrino per generation,  $\nu_R$ , is introduced. The symmetry of lepton and quark contents and the hypercharge symmetry in Eq. (14) are re-established. At the underlying level of preleptons defined in Eqs. (17) and (19), one has the presymmetry transformations that exchange topo-

logical prelepton and quark fields in a Lagrangian where Majorana mass terms are absent since all preleptons have electroweak charges.

By introducing the second RH neutrino for each generation of leptons and quarks, which breaks presymmetry, the second hierarchy,  $M'_D \ll M_D$ , is understood naturally since if this new parameter is set equal to zero, the presymmetry involving the first RH neutrino is recovered.

The last hierarchies,  $(M_D M_R^{-1} M_D^T, M' M_R^{-1} M'^T, M_D M_R^{-1} M'^T) \ll M'_D$  (entailing  $M_D, M' \ll M_R$ ) and  $M'_R \ll M'_D$ , mimic the low scale pseudo-Dirac scenario. If the small ratios of mass parameters are neglected, for all practical purposes the first RH neutrino is decoupled from the low energy model. As such, the symmetry of lepton and quark contents, the symmetric charge relations in Eq. (14) and in Eqs. (17) and (19) at the underlying level, as well as presymmetry, now engage  $\nu'_R$  instead of  $\nu_R$ . In this way, the residual presymmetry connecting  $\nu'_R$  makes the low scale model more symmetric. Thus all the hierarchies are natural in the sense of t' Hooft.

## 5. Phenomenological Implications of the Model

In order to have light neutrinos of adulterated Dirac type, our extension of the SM assumes two RH neutrinos in each generation and the hierarchy of masses given in Eq. (12). As discussed above, the first choice is to decouple through the extended seesaw mechanism the original RH neutrinos from the others by making them much heavier than the other mass parameters and ratios of mass parameters. In a second step, the light neutrino masses are effectively controlled by the new Dirac mass of the extra RH neutrinos. This requires a hierarchy between the extra Dirac masses and the other mass parameters and ratios of mass parameters. Only this second hierarchy gives rise to the light pseudo-Dirac neutrinos.

The Dirac mass  $M_D$  is supposed to be of order the charged lepton mass. In the approximation of one generation, if a mass  $M_D = \mathcal{O}(1 \text{ MeV})$  is for the charged lepton of first generation and the upper bounds

$$M_{LL}, M'_{RR} \leq \mathcal{O}(10^{-9} \text{ eV}) \quad (29)$$

obtained from data analyzes on solar neutrino oscillations<sup>28,29</sup> are considered, then Eq. (9) leads to an energy threshold for the type-I seesaw equal to  $M_R = \mathcal{O}(10^{12} \text{ GeV})$ . Besides, using the experimental data on neutrino mass,<sup>30</sup>

$$M'_{LR} = \mathcal{O}(10^{-1} \text{ eV}), \quad (30)$$

the indicative values for the parameters in the model become

$$\begin{aligned} M_R &\geq \mathcal{O}(10^{12} \text{ GeV}), & M_D &= \mathcal{O}(1 \text{ MeV}), & M'_D &= \mathcal{O}(10^{-1} \text{ eV}), \\ M'_R &\leq \mathcal{O}(10^{-9} \text{ eV}), & M' &\leq \mathcal{O}(1 \text{ MeV}), \end{aligned} \quad (31)$$

which are consistent with the hierarchy of masses

$$M_{LL}, M'_{RR} \ll M'_{LR} \ll M_D \ll M_R \quad (32)$$

adopted in the model, so ratifying the Dirac type assumed for light neutrinos.

Thus the parameter region being considered excludes the pseudo-Dirac limit, but not the Dirac character for light neutrinos. Their masses or Yukawa couplings may have exceptionally small values because of the adulterant character of RH partners. Also, there is consistency between this Dirac picture and the vanishing of the Majorana mass  $M'_R$  assumed above. Besides, the Dirac nature of lighter neutrinos forbids the neutrinoless double-beta decay, in accordance with recent precision experiments.<sup>14–16</sup> Even more, no significant departures from the SM predictions are expected at the TeV region, leading to substantive tensions with models which assume extensions of the gauge and Higgs sectors with breaking scales at the TeV range. Thus, the model can be tested through the successes of the SM and the Dirac nature of light neutrinos. Although we do not address here the phenomenon of dark matter, we note that our model can be extended to accommodate its particles, for instance, by adding extra sterile neutrinos.

Yet, the model maintains the expectations of the high scale type-I seesaw mechanism.<sup>10–13</sup> This includes the new physics to be introduced with its energy threshold  $\mathcal{O}(10^{12} \text{ GeV})$ , required to solving the problem of naturalness generated by the quantum corrections to the Higgs boson mass. In fact, the type-I seesaw is natural up to  $M_R = \mathcal{O}(10^7 \text{ GeV})$ ,<sup>20</sup> but demands an unnatural fine-tuning cancellation for  $M_R = \mathcal{O}(10^{12} \text{ GeV})$ . The only best known way of persisting is through supersymmetry. This can be implemented in stages, starting with the non-supersymmetric low-energy theory,<sup>21</sup> although the high seesaw threshold calls for the supersymmetrization of the Higgs, electroweak and fermionic sectors. The no observation of supersymmetric phenomena may imply that the quantum corrections to the Higgs mass would be suppressed by another kind of new physics.

On the other hand, the model is crucially based on the hypothesis of a symmetry of quark and lepton contents, which, however, is validated by presymmetry. Light Dirac neutrinos are then predictions of presymmetry in the SM extended with two RH neutrinos per generation, implemented with a high scale seesaw mechanism. Moreover, presymmetry of the SM with adulterated Dirac neutrinos appears as a residue after removal of the heavy RH neutrinos of Majorana type. Thus, the signatures of presymmetry are also marks of the proposed SM with adulterated Dirac neutrinos. Besides light Dirac neutrinos, they include explanations of the fractional charge of quarks and quark–lepton charge relations, understanding of the equality between the number of generations and the number of quark colors, accounting for the topological charge conservation in quantum flavor dynamics, and elucidation of the charge quantization and the no observation of fractionally charged hadrons.<sup>18,19</sup>

## 6. Conclusions

In the scenario of the SM extended with one RH neutrino per generation, a simple paradigm for understanding the small value of neutrino masses compared to the

charged leptons is the type-I seesaw mechanism where RH neutrinos have Majorana masses in addition to Dirac masses. A distinguishing feature of this mechanism is the Majorana nature of light neutrinos, which, however, is not favored by recent experimental data on double-beta decay of nuclei. Moreover, the current experimental situation shows an agreement with the SM predictions well above the TeV, apart from the presumed Dirac neutrinos with small masses for which it gives no explanation.

Assuming no serious departures from the SM expectations at the TeV range and the Dirac character of light neutrinos, recently we proposed an extended seesaw in which two RH neutrinos per generation are added, implemented with the hypothesis of the symmetry of lepton and quark contents in order to restrain the number of RH neutrinos from freedom, produce Dirac neutrinos and naturally give them tiny masses. The first one is the usual RH neutrino which re-establishes the correspondence between quarks and leptons at high energies with weak couplings having order of magnitudes as those of its weak charged partner and a Majorana mass term whose coupling is assumed to be large, as in the canonical high-scale type-I seesaw scenario. The second RH neutrino, which breaks the quark-lepton symmetry founded with the first one, has small masses and couplings, as explained by the 't Hooft's naturalness criterion applied to this symmetry of contents. The first RH neutrino is decoupled at the high scale, while the second one survives down to the low scale to pair off in a Dirac-like fashion with the corresponding LH neutrino, driving its pattern of the symmetry of fermionic content. These symmetries of particle content were only regarded as guidelines to the choice of parameters since they cannot be understood as symmetry transformations that exchange lepton and quark fields in the Lagrangian of the model. It was supposed, however, that the proper symmetry to take on has to be hidden in the SM with RH neutrinos itself.

From another viewpoint, presymmetry was assumed as a symmetry that underlies the SM extended with three RH neutrinos having Dirac mass terms, so restoring lepton-quark symmetry of particle content, unifying the electroweak properties of leptons and quarks, and explaining the observed charge relations and chiral structure of weak interactions. However, this scenario cannot account naturally for the smallness of Dirac neutrino mass terms relative to those of charged leptons. Besides, if it is pointed out that the  $B - L$  symmetry is an accidental symmetry of the SM and that presymmetry is a hidden electroweak symmetry at a bare level, the inclusion of Majorana mass terms for neutrinos seems natural. Thus, both Dirac and Majorana mass terms are included in the minimal SM extension of only three RH neutrinos, explicitly breaking the conservation of the lepton number. Presymmetry still appears as a basic symmetry of the model with neutrinos having generic mass terms, since it is an extra enhanced symmetry established at the bare step; such presymmetry transformations perform at the underlying level of topological bare states which have the same electroweak charges and no Majorana mass terms. Yet, the tiny mass of Dirac-like neutrinos respect to charged leptons still remains unnatural, so that the addition of just three RH neutrinos with generic masses to

the SM is not enough for understanding such smallness.

The addition of a second RH neutrino per generation, however, provides the seed for the expected light Dirac neutrinos. We have shown that presymmetry defines properly the symmetry transformations required by the symmetry of lepton and quark contents and the assumed sequence of hierarchies. This means that the extra Dirac and Majorana masses are not free parameters, independent of the first RH neutrinos. We emphasize that these are not made small by fine-tuning. Their smallness compared to the first RH neutrinos are guaranteed by the presymmetry defined with the usual RH neutrinos, at the high-energy seesaw scale. The 't Hooft's argument of naturalness for the small values of the Dirac and Majorana mass terms of the extra RH neutrinos in the Lagrangian relies on this presymmetry with the first RH neutrinos; as the couplings of the extra RH neutrinos tend to zero, the underlying theory only involving the first RH neutrinos becomes more symmetric. This symmetry guarantees the quantum corrections of such parameters to be proportional to the parameters themselves and its interplay with the seesaw mechanism leading to the low-energy effective theory with the original RH neutrinos decoupled only introduces negligible corrections to the mass parameters. In particular, the smallness of the Dirac mass of neutrinos compared to the charged leptons is stable.

Now a low scale Dirac scenario with symmetry of particle content and small neutrino masses appears natural, satisfying 't Hooft's naturalness conditions. But, they involve the additional and not the standard RH neutrinos, which are decoupled. The claim is that the SM extended with extra RH neutrinos and implemented with the seesaw mechanism and presymmetry, or the presymmetry model implemented with extra RH neutrinos and the seesaw mechanism, makes natural the existence of light Dirac-like neutrinos. Neutrinos with extremely small masses are predicted to be of adulterated Dirac nature in the sense that the ordinary RH components are replaced by the almost inert extra ones. Besides, the parameter region considered in this approach makes irrelevant to low energy processes the perturbation of the seesaw mechanism on a description given in terms of light Dirac neutrinos, foreseeing that experiments will not have sensitiveness to the Majorana character of neutrinos predicted by the seesaw mechanism, as in the case of the neutrinoless double-beta decay.

On the other hand, the signatures of presymmetry are also noticeable features of the model of Dirac neutrinos, such as the topological character of fractional charges and the relationship between the number of generations and the number of quark colors. Although there are no hard predictions for the masses and mixing of light neutrinos, the model does provide a new line of physics beyond the SM for exploration.

Nevertheless, the high energy seesaw threshold established by our approach raises the issue on the naturalness of the renormalization of the Higgs mass due to the quantum corrections introduced by the new physics states associated with the seesaw. The best known way to solve it is via supersymmetry. An eventual tension between experimental data and supersymmetry expectations at the TeV range



to be tested by the LHC may imply that the solution to this naturalness problem would be given by another, still unknown new physics.

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